

Multi-User Diversity with Random Number of Users

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Abstract

Multi-user diversity is considered when the number of users in the system is random. The complete monotonicity of the error rate as a function of the (deterministic) number of users is established and it is proved that randomization of the number of users always leads to deterioration of average system performance at any average SNR. Further, using stochastic ordering theory, a framework for comparison of system performance for different user distributions is provided. For Poisson distributed users, the difference in error rate of the random and deterministic number of users cases is shown to asymptotically approach zero as the average number of users goes to infinity for any fixed average SNR. In contrast, for a finite average number of users and high SNR, it is found that randomization of the number of users deteriorates performance significantly, and the diversity order under fading is dominated by the smallest possible number of users. For Poisson distributed users communicating over Rayleigh faded channels, further closed-form results are provided for average error rate, and the asymptotic scaling law for ergodic capacity is also provided. Simulation results are provided to corroborate our analytical findings.

Index Terms

Multi-user Diversity, Completely Monotone Functions, Stochastic Ordering.

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I. INTRODUCTION

Point to point diversity combining schemes aim to mitigate the effects of fading in a wireless channel. In contrast, for multi-user systems another form of diversity termed *multi-user diversity* (MUD) is available, which thrives on the randomness of the user fading channels [1]. The key idea is to provide channel access to the user with the best channel at any instant of time. This has been shown to be optimal for both uplink [2] and downlink [3] scenarios.

In the literature, MUD has been studied for the case of deterministic number of users only. Since the number of users is randomly varying in practice, it is of interest to consider MUD for this case as well. For example, cell phone users have longer voice calls while channel access for data communication is very short [4]. The probability of a cell phone user requesting data communication is very low, and bursty data requests such as stocks, weather and email lead to very short channel access times. This suggests that the number of users actively contending for channel access across time is random. Additionally, schemes in which a user is allowed to feedback its channel estimate to request channel access, when it is larger than a predefined threshold [5]–[7], also lead to a random number of users. Even in common scenarios where the fluctuations in the number of users is slower than the rapidity of channel fading, averaging error rates, or ergodic capacity, with respect to the user distribution results in meaningful system-level performance measures.

In this paper, we analyze the performance of MUD systems with random number of users for the first time in the literature. In Section II the instantaneous SNR distribution of the best user chosen from a random set of users is derived for arbitrary fading and user distributions, and the mathematical preliminaries are presented. In Section III, the error rate averaged across fading with deterministic number of users is shown to be a completely monotonic function of the number of users N . Further, we also prove that the ergodic capacity of a MUD system with a deterministic number of users has a completely monotone derivative with respect to the number of users. These structural results of performance for a deterministic number of users are then used

to prove facts about the random number of users case. The first of these is that randomization of the number of users results in the deterioration of average performance measured in terms of either error rate or ergodic capacity, by using Jensen's inequality. In Section IV we introduce a framework in which different user distributions can be compared through the so called Laplace transform partial ordering of the number of users which is a particular stochastic order [8]. In Section V we derive the diversity order of a MUD system with random number of users and show that it is determined by the minimum possible number of users. In Section VI-A, expressions for outage for any fading distribution with Poisson user distribution are derived. For when the user distribution is Poisson distributed, Jensen's inequality for error rate is proved to be asymptotically tight in the average number of users in Section VI-B. For the special case when the number of users is Poisson distributed and when the user channel is Rayleigh faded, a closed-form expression for the error rate is derived in Section VI-C. The scaling of ergodic capacity with the average number of users is also provided for this case in Section VI-D. Poisson user distribution without allowing the number of users to be zero is considered in Section VI-E. Section VII corroborates our analytical results with simulations and Section VIII concludes the paper.

Here are some remarks on notations used in this work. Asymptotic equivalence $\tau(x) \sim g(x)$ as $x \rightarrow a$ means that $\lim_{x \rightarrow a} \tau(x)/g(x) = 1$, and $\tau(x) = O(g(x))$ as $x \rightarrow a$ means that $\limsup_{x \rightarrow a} |\tau(x)/g(x)| < \infty$. In this paper, we consider $a = 0$ or $a = \infty$. $\Pr[\cdot]$ is the probability of an event, $E_{\mathcal{X}}[f(\mathcal{X})]$ is the expectation of the function $f(\cdot)$ over the distribution of the random variable \mathcal{X} , and $\log(\cdot)$ is logarithm to base e .

II. SYSTEM MODEL AND MATHEMATICAL PRELIMINARIES

We consider an uplink MUD system with one base station (BS) and multiple users. Without loss of generality, both the BS and the users are assumed to have a single antenna. The received signal at the BS from the n^{th} user can be expressed as,

$$y_n = \sqrt{\rho} h_n x_n + w_n, \quad n = 0, 1, \dots, \mathcal{N}, \quad (1)$$

where the number of users \mathcal{N} is assumed to be a random variable with a discrete non-negative integer distribution. In Section VI-E we address the implications on the performance metrics of allowing the probability of $\mathcal{N} = 0$ to be positive, leading to possibly no users and no transmission. When addressing the deterministic number of users case, we will set $\mathcal{N} = N$, where N is a realization of the random variable \mathcal{N} . A homogeneous MUD system is assumed where the average received power at the BS, ρ , is identical across all users. The symbol h_n denotes the channel coefficient, x_n the transmitted symbol, and w_n the additive white Gaussian noise (AWGN) corresponding to the n^{th} user. The channel is assumed to satisfy $\mathbb{E}[|h_n|^2] = 1$ for all n and to be independent and identically distributed (i.i.d.) across all users. The transmitted symbols satisfy $\mathbb{E}[|x_n|^2] = 1$.

The channel gain of the n^{th} user at the BS, prior to selection, can be expressed as $\gamma_n = |h_n|^2$, and the selected user has a channel gain denoted by $\gamma^* = |h_*|^2$, where $|h_*|^2 = \max_n \{|h_n|^2\}$. Note that γ^* is a random variable that depends on the random variables \mathcal{N} , and $|h_n|^2$, $n = 0, 1, \dots, \mathcal{N}$.

Define $F_{\gamma_n}(x)$ as the cumulative distribution function (CDF) of the channel gain of the n^{th} user γ_n . Since the fading channels across all users are assumed to be i.i.d., we drop the index n and define $F_\gamma(x) := F_{\gamma_n}(x)$. Recalling that the total number of users \mathcal{N} is a random variable, the CDF of the channel gain of the selected user, conditioned on $\mathcal{N} = N$, can be written as:

$$F_{\gamma^*}(x|\mathcal{N} = N) = F_\gamma^N(x), \quad (2)$$

where the N^{th} power is obtained due to the i.i.d. assumption of the N user channels. The CDF of the channel gain of the best user selected from a random set of users can be obtained by averaging (2) with respect to the distribution of \mathcal{N} :

$$F_{\gamma^*}(x) = \mathbb{E}_{\mathcal{N}} [F_\gamma^{\mathcal{N}}(x)] = \sum_{k=0}^{\infty} \Pr[\mathcal{N} = k] F_\gamma^k(x) = U_{\mathcal{N}}(F_\gamma(x)) \quad (3)$$

where $U_{\mathcal{N}}(t) = \sum_{k=0}^{\infty} \Pr[\mathcal{N} = k] t^k$, $0 \leq t \leq 1$, is the probability generating function (PGF) of random variable \mathcal{N} . From (3) it can be seen that for any fading channel distribution and any

non-negative integer distribution on the number of users, the CDF of the best user's channel gain at the BS can be easily obtained.

We now survey some mathematical preliminaries that will be useful throughout. A function $\tau(x) : \mathbb{R}^+ \rightarrow \mathbb{R}$ is *completely monotonic* (*c.m.*) if its derivatives alternate in sign [8], i.e.,

$$(-1)^k \frac{d^k \tau(x)}{dx^k} \geq 0, \quad \forall x, \quad k = 0, 1, 2, \dots, \quad (4)$$

where $d^0 \tau(x)/dx^0 = \tau(x)$ by definition. Due to a celebrated theorem by Bernstein [8], an equivalent definition for *c.m.* is that it is a positive mixture of decaying exponentials. In other words, we have the Bernstein's representation $\tau(x) = \int_0^\infty e^{-sx} d\psi(s)$ for some nondecreasing function $\psi(s)$. In this paper, we are sometimes interested in *c.m.* functions on integers, which are nothing but sequences obtained by sampling *c.m.* functions as defined by (4). We are also interested in functions whose first-order derivatives satisfy (4), which are said to have a completely monotone derivative (*c.m.d.*). Even when the variable x is naturally an integer (such as the number of users), we will sometimes treat it as a real number, since we will be primarily interested in the asymptotic properties of $\tau(x)$.

A function $\psi(s)$ is *regularly varying* with exponent $\mu \neq 0$ at $s = \infty$ if it can be expressed as $\psi(s) = s^\mu l(s)$ where $l(s)$ is slowly varying and by definition satisfies $\lim_{s \rightarrow \infty} l(\kappa s)/l(s) = 1$ for $\kappa > 0$. Regular (slow) variation of $\psi(s)$ at $s = 0$ is equivalent to regular (slow) variation of $\psi(1/s)$ at ∞ . Intuitively, regular variation captures polynomial-like behavior near the origin or at infinity. The Tauberian theorem for Laplace transforms, whose proof can be found in [9], applies to *c.m.* functions and states that $\tau(x)$ is regularly varying at $x = \infty$ if and only if $\psi(s)$ is regularly varying at $s = 0$:

Theorem 1: If a nondecreasing function $\psi(s) \geq 0$ defined on $s \geq 0$ has Laplace transform $\tau(x) = \int_0^\infty e^{-sx} d\psi(s)$ for $x \geq 0$, and $l(s)$ is slowly varying at $s = 0$ (or $s = \infty$), the relations $\psi(s) \sim s^\mu l(s)$ as $s \rightarrow 0$ (or $s \rightarrow \infty$) and $\tau(x) \sim \Gamma(\mu + 1)x^{-\mu}l(x^{-1})$ as $x \rightarrow \infty$ (or $x \rightarrow 0$) imply each other, where $\mu \in \mathbb{R}$.

One useful property given in [10, p.27] is that

$$\int_0^t s^\mu l(s) ds \sim l(t) \int_0^t s^\mu ds = \frac{1}{\mu+1} t^{\mu+1} l(t) \quad (5)$$

as $t \rightarrow 0$, with $\mu > -1$ for $l(s)$ slowly varying at $s = 0$.

In this paper, we are interested in studying average error rates, and capacities averaged across both the channel distribution, and the number of users. The expression $\bar{P}_e(\rho, N)$ represents the error rate of a MUD system with a deterministic number of users N , that is averaged with respect to the distribution of the fading channel. The expression $E_{\mathcal{N}} [\bar{P}_e(\rho, \mathcal{N})]$ represents the average error rate of a MUD system with a random number of users, which is averaged with respect to the distribution of the number of users *and* the fading channels.

III. PROPERTIES OF THE AVERAGE ERROR RATE AND ERGODIC CAPACITY

A. Average Error Rate

In this section, we first prove that the average error rate of a MUD system, with a deterministic number of users N , is a *c.m.* function of N , under general conditions. This will be used to infer about the behavior of the average error rate when a random number of users is considered, in Section IV.

The error rate of a MUD system with a deterministic number of users N and average SNR ρ is given by,

$$\bar{P}_e(\rho, N) = \int_0^\infty P_e(\rho x) dF_\gamma^N(x) \quad (6)$$

where $P_e(\rho x)$ is the instantaneous error rate over an AWGN channel for an instantaneous SNR ρx of the best user. Often, the instantaneous error rate is assumed to have the form $P_e(\rho x) = \alpha e^{-\eta \rho x}$ or $P_e(\rho x) = \alpha Q(\sqrt{\eta \rho x})$, where α and η can be chosen to capture different modulations [11]. To represent (6) in terms of the CDF $F_\gamma(x)$, rather than the probability density function (PDF), we left it as a Stieltjes integral [12] even though it can also be expressed in terms of the PDF $f_\gamma(x)$ using $dF_\gamma^N(x) = N F_\gamma^{N-1}(x) f_\gamma(x)$. In what follows, we will study the sequence $\bar{P}_e(\rho, N)$ as a

function of the integer variable N . Since we are ultimately interested in the asymptotic behavior of this sequence, we will also consider (6) with N being a real number.

We begin by proving that $\bar{P}_e(\rho, N)$ is a *c.m.* function of N not just for $P_e(\rho x)$ in the forms of exponential function and Q function, but for any instantaneous error rate function. In other words, we only assume $P_e(\rho x)$ is decreasing in x for any $\rho > 0$. Defining $B(x) = -dP_e(x)/dx$, after integrating (6) by parts, the k^{th} derivative of $\bar{P}_e(\rho, N)$ can be written as,

$$\frac{\partial^k \bar{P}_e(\rho, N)}{\partial N^k} = \rho \int_0^\infty B(\rho x) F_\gamma^N(x) [\log(F_\gamma(x))]^k dx. \quad (7)$$

Since $P_e(\rho x)$ is decreasing, and $\log(F_\gamma(x)) \leq 0$ we see that (7) satisfies the definition in (4). In particular, $\bar{P}_e(\rho, N)$ being a *c.m.* function means that (7) is negative for $k = 1$ and positive for $k = 2$, and consequently $\bar{P}_e(\rho, N)$ is a convex decreasing function of N . For when the number of users in the system is random, by applying Jensen's inequality for convex functions, we have,

$$E_{\mathcal{N}} [\bar{P}_e(\rho, \mathcal{N})] \geq \bar{P}_e(\rho, \lambda), \quad (8)$$

where $\lambda := E[\mathcal{N}]$. Therefore, randomization of the number of users always deteriorates the average error rate performance of a MUD system.

To establish the complete monotonicity of $\bar{P}_e(\rho, N)$ as a function of N , we only used the fact that the instantaneous error rate $P_e(\rho x)$ in (6) is a decreasing function of x for $\rho > 0$, which always holds. This *c.m.* property will be used to stochastically order user distributions in Section IV.

B. Ergodic Capacity

The ergodic capacity for the deterministic number of users system can be expressed as,

$$\bar{C}(\rho, N) = \int_0^\infty \log(1 + \rho x) dF_\gamma^N(x) = \rho \int_0^\infty \frac{1 - F_\gamma^N(x)}{1 + \rho x} dx. \quad (9)$$

where we use integration by parts, and assume that $F_\gamma(x)$ satisfies $\lim_{x \rightarrow \infty} \log(1 + \rho x)(1 - F_\gamma^N(x)) = 0$, for all $N \geq 0$.

It can be seen that $\overline{C}(\rho, N)$ has a completely monotonic derivative since,

$$\frac{\partial^{k+1}\overline{C}(\rho, N)}{\partial N^{k+1}} = -\rho \int_0^\infty \frac{F_\gamma^N(x) [\log(F_\gamma(x))]^{k+1}}{1 + \rho x} dx. \quad (10)$$

alternates in sign as k is incremented. This establishes that $\overline{C}(\rho, N)$ has a completely monotonic derivative, provided that the fading distribution satisfies the mild assumption $\lim_{x \rightarrow 0} \log(1 + \rho x)(1 - F_\gamma^N(x)) = 0$ for all $N \geq 0$, as assumed after (9). This assumption holds for all distributions with exponential or power law tails, which is the case for all fading distributions in wireless communications. Using (10) with $k = 0, 1$, it is seen that $\overline{C}(\rho, N)$ is concave increasing function of N . Applying Jensen's inequality for concave functions, we have

$$\mathbb{E}_N [\overline{C}(\rho, \mathcal{N})] \leq \overline{C}(\rho, \lambda). \quad (11)$$

Therefore, similar to the error rate metric, randomization of N will always hurt the average ergodic capacity of a MUD system. The *c.m.d.* property of $\overline{C}(\rho, N)$ will be used in the following section discussing the stochastic Laplace transform ordering of user distributions.

IV. LAPLACE TRANSFORM ORDERING OF USER DISTRIBUTIONS

In this section we introduce Laplace transform (LT) ordering, a tool to compare the effect that different user distributions has on the error rate, and ergodic capacity averaged across user and channel distributions. Stochastic ordering of random variables, of which LT ordering is a special case, is a branch of probability theory and statistics which deals with binary relations between random variables [8], [13].

Let \mathcal{X} and \mathcal{Y} be non-negative random variables. \mathcal{X} is said to be less than \mathcal{Y} in the LT order (written $\mathcal{X} \leq_{Lt} \mathcal{Y}$), if $\mathbb{E}[e^{-s\mathcal{X}}] \geq \mathbb{E}[e^{-s\mathcal{Y}}]$ for all $s > 0$. An important theorem found in [8], and [13] is given next:

Theorem 2: Let \mathcal{X} and \mathcal{Y} be two random variables. If $\mathcal{X} \leq_{Lt} \mathcal{Y}$, then, $\mathbb{E}[\psi(\mathcal{X})] \geq \mathbb{E}[\psi(\mathcal{Y})]$ for all *c.m.* functions $\psi(\cdot)$, provided the expectation exists. Moreover, the reverse inequality

$E[\psi(\mathcal{X})] \leq E[\psi(\mathcal{Y})]$ holds for all $\psi(\cdot)$ with a completely monotone derivative, provided the expectation exists.

It follows that if two user distributions satisfy $\mathcal{N}_1 \leq_{Lt} \mathcal{N}_2$, then for all average SNR ρ

$$\begin{aligned} E_{\mathcal{N}_1} [\bar{\mathbf{P}}_e(\rho, \mathcal{N}_1)] &\geq E_{\mathcal{N}_2} [\bar{\mathbf{P}}_e(\rho, \mathcal{N}_2)] , \\ E_{\mathcal{N}_1} [\bar{\mathbf{C}}(\rho, \mathcal{N}_1)] &\leq E_{\mathcal{N}_2} [\bar{\mathbf{C}}(\rho, \mathcal{N}_2)] . \end{aligned} \quad (12)$$

To rephrase (12), if the number of users \mathcal{N} is from a distribution that is dominated by another distribution in the Laplace transform sense, then both the average error rate and capacity are respectively ordered at all average SNR ρ .

The LT ordering of discrete random variables can also be expressed in terms of the ordering of their PGFs. By defining $t := e^{-s}$, one can rewrite $E[e^{-s\mathcal{X}}] \geq E[e^{-s\mathcal{Y}}]$ for $s \geq 0$ as $E[t^{\mathcal{X}}] \geq E[t^{\mathcal{Y}}]$ for $0 \leq t \leq 1$, which is the same as $U_{\mathcal{X}}(t) \geq U_{\mathcal{Y}}(t)$, $0 \leq t \leq 1$, where we recall that $U_{\mathcal{X}}(t) = E[t^{\mathcal{X}}]$ represents the PGF of the discrete random variable \mathcal{X} .

To provide examples of random variables that are LT ordered, consider Poisson random variables \mathcal{X} and \mathcal{Y} with means λ and μ respectively, such that $\lambda \leq \mu$. It is straightforward to show that for this case $e^{\lambda(t-1)} = E[t^{\mathcal{X}}] \geq E[t^{\mathcal{Y}}] = e^{\mu(t-1)}$, for $0 \leq t \leq 1$, implying that $\mathcal{X} \leq_{Lt} \mathcal{Y}$. If \mathcal{X} and \mathcal{Y} are geometric distributed with probability of success on each trial p_1 and p_2 respectively, such that $p_1 \leq p_2$, then $\mathcal{Y} \leq_{Lt} \mathcal{X}$ since $p_2/(1 - (1 - p_2)t) = E[t^{\mathcal{Y}}] \leq E[t^{\mathcal{X}}] = p_1/(1 - (1 - p_1)t)$ for $0 \leq t \leq 1$. Similarly, for \mathcal{X} being Poisson distributed with parameter λ and \mathcal{Y} being geometric distributed with parameter $p = 1/(1 + \lambda)$, so that $E[\mathcal{X}] = E[\mathcal{Y}] = \lambda$, it can be once again shown that $E[t^{\mathcal{Y}}] \leq E[t^{\mathcal{X}}]$ for $0 \leq t \leq 1$, establishing $\mathcal{Y} \leq_{Lt} \mathcal{X}$. From this latter result, one can conclude that Poisson offers a better user distribution than geometric distribution for a fixed average number of users at all average SNR ρ , from both error rate and capacity points of view.

V. HIGH SNR ANALYSIS AND DIVERSITY ORDER

In this section we analyze the average error rate at high SNR under general assumptions on the user distribution and fading channel distribution. In the following, we will assume that the

fading distribution $F_\gamma(x)$ is regularly varying with exponent d at $x = 0$ (typically $F_\gamma(x) = O(x^d)$ as $x \rightarrow 0$), which is true for many fading distributions including Rayleigh ($d = 1$), Nakagami- m ($d = m$) and Ricean ($d = 1$) [14].

Theorem 3: Let $F_\gamma(x)$ be regularly varying at $x = 0$ with exponent $d > 0$, and $\mathcal{N} \in \{k_0, k_0 + 1, \dots\}$ represent the range of the number of users random variable. Then the high-SNR asymptotic average error rate is given by

$$\mathbb{E}_{\mathcal{N}} [\bar{P}_e(\rho, \mathcal{N})] \sim \Pr[\mathcal{N} = k_0] C_1 F_\gamma^{k_0}(C_2 \rho^{-1}) \quad (13)$$

as $\rho \rightarrow \infty$, where C_1 and C_2 are constants given by $C_1 = \alpha \Gamma(k_0 d + 1)$, $C_2 = \eta^{-1}$ when $P_e(\rho x) = \alpha e^{-\eta \rho x}$ and $C_1 = \alpha \Gamma(k_0 d + 1/2)/(2\sqrt{\pi})$, $C_2 = 2\eta^{-1}$ when $P_e(\rho x) = \alpha Q(\sqrt{\eta \rho x})$.

Proof: See Appendix A. ■

From (13) it is straightforward to show that the diversity order of the MUD system with random number of users is given by $k_0 d$. This follows from (13) and the regular variation assumption on $F_\gamma(x)$, so that $F_\gamma^{k_0}(C_2 \rho^{-1}) = O(\rho^{-k_0 d})$ as $\rho \rightarrow \infty$.

VI. POISSON DISTRIBUTED \mathcal{N}

Consider a MUD system which contains a large number of users. Suppose each user is active with a small probability. In such a system, as the number of users increases, the user distribution will approximate the Poisson. In this section we analyze the system when \mathcal{N} is Poisson distributed with parameter λ .

A. Outage Probability and Its Asymptotic Behavior for Large λ

When \mathcal{N} is Poisson distributed with parameter λ , the probability of outage with a threshold x can be expressed as,

$$F_{\gamma^*}(x) = \Pr[\gamma^* \leq x] = \sum_{k=0}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!} F_\gamma^k(x) = e^{-\lambda(1-F_\gamma(x))} I[x \geq 0] \quad (14)$$

where $I[\cdot]$ is the indicator function. Equation (14) implies that as the average number of users increases, the outage probability decreases for any distribution on γ_n , the channel gain of the user fading channel.

In what follows, we show that for large λ the outage behavior is dependent on $F_\gamma(x)$ only through its tail behavior. In fact, it is possible to show that there exist normalizing and shift functions $a(\lambda)$ and $b(\lambda)$ such that the probability $\Pr[(\gamma^* - b(\lambda))/a(\lambda) \leq x]$ for large λ is

$$\lim_{\lambda \rightarrow \infty} F_{\gamma^*}(a(\lambda)x + b(\lambda)) = \exp(-e^{-x}), \quad -\infty < x < \infty \quad (15)$$

which is known as the Gumbel distribution [15]. Using (14), sufficient and necessary conditions for (15) are clearly that $\lim_{\lambda \rightarrow \infty} \lambda(1 - F_\gamma(a(\lambda)x + b(\lambda))) = e^{-x}$. In [15, p.300] it is shown that $a(\lambda) = [\lambda f_\gamma(b(\lambda))]^{-1}$, $b(\lambda) = F_\gamma^{-1}(1 - 1/\lambda)$, satisfy this condition for many distributions including Rayleigh, Nakagami- m and Ricean. Also, it can be seen that the asymptotic CDF of the Poisson number of users case for large λ has the same form (Gumbel distribution) as the asymptotic CDF of the deterministic number of users case for large N .

B. Average Error Rate

In our outage analysis for the Poisson number of users case, we showed that the outage probability in (14) for large λ approaches the Gumbel distribution, which is also the asymptotic distribution obtained for a deterministic number of users case. Therefore, even though we saw that randomization always deteriorates performance, for large average number of users it should approximately yield the same performance as the deterministic case. This amounts to the tightness of Jensen's inequality for the Poisson users case.

We now provide sufficient conditions for Jensen's inequality involving $\bar{P}_e(\rho, N)$ in (12) to be asymptotically tight in λ . Recall that $\bar{P}_e(\rho, N)$ is the error rate averaged over the channel distribution for deterministic number of users N . To this end, we use the results in [16, Theorem 2.2] which were derived in a networking context for arbitrary *c.m.* functions.

Theorem 4: Let $\bar{P}_e(\rho, N)$ be *c.m.* and regularly varying at $N = \infty$ and consider the error

rate averaged across the channel and the users $E_{\mathcal{N}} [\bar{P}_e(\rho, \mathcal{N})]$, where \mathcal{N} is a Poisson distributed random variable with mean λ . Then,

$$E_{\mathcal{N}} [\bar{P}_e(\rho, \mathcal{N})] = \bar{P}_e(\rho, \lambda) + O(\bar{P}_e(\rho, \lambda)/\lambda) \quad (16)$$

as $\lambda \rightarrow \infty$.

Equation (16) shows that as $\lambda \rightarrow \infty$, the difference between the error rate averaged across the user distribution and the error rate evaluated at the average number of users vanishes as λ tends to ∞ . This implies that for sufficiently large λ the performance of the MUD systems with random number of users will be almost equal to the performance of the MUD systems with a deterministic number of users with the number of users equal to λ .

To apply Theorem 4 we require $\bar{P}_e(\rho, N)$ to be *c.m.* and regularly varying. We have already shown that $\bar{P}_e(\rho, N)$ is always completely monotonic in N . Next, we provide the conditions under which $\bar{P}_e(\rho, N)$ is a regularly varying function of N . Consider

$$\bar{P}_e(\rho, N) = \rho \int_0^\infty B(\rho x) e^{N \log(F_\gamma(x))} dx \quad (17)$$

where $B(\cdot)$ is defined as $B(x) = -dP_e(x)/dx$. Now, setting $u := -\log(F_\gamma(x))$, and integrating by substitution we have,

$$\bar{P}_e(\rho, N) = \rho \int_0^\infty \frac{B(\rho F_\gamma^{-1}(e^{-u})) e^{-u} e^{-uN} du}{f_\gamma(F_\gamma^{-1}(e^{-u}))}, \quad (18)$$

where $F_\gamma^{-1}(x)$ is the inverse CDF and $f_\gamma(x)$ is the PDF of γ_n . We now establish the sufficient conditions for $\bar{P}_e(\rho, N)$ to be a regularly varying function of N :

Theorem 5: If $\bar{P}_e(\rho, N)$ is *c.m.* in N , a sufficient condition for it to be regularly varying at $N = \infty$ is that, $t(u) := \rho(B(\rho F_\gamma^{-1}(e^{-u})) e^{-u}) / (f_\gamma(F_\gamma^{-1}(e^{-u})))$ is regularly varying at $u = 0$.

Proof: By comparing the representation of $\bar{P}_e(\rho, N)$ in (18) with the Bernstein's representation of *c.m.* functions discussed after (4), it can be seen that (18) can be represented as the Laplace transform of $t(u)$. Using Theorem 1, the proof follows. ■

Theorem 5 shows that for the conclusions of Theorem 4 to hold (i.e., Jensen's inequality to be asymptotically tight), the CDF of the single-user channel $F_\gamma(x)$, and the error rate expression $P_e(\rho x)$ have to jointly satisfy the regular variation condition given in Theorem 5. Next, we examine whether this condition holds for commonly assumed instantaneous error rates $P_e(\rho x)$ with γ_n being exponentially distributed. For the case of $P_e(\rho x) = \alpha e^{-\eta \rho x}$, we have $t(u) = \alpha \rho (1 - e^{-u})^{\eta \rho - 1} e^{-u}$, which satisfies $\lim_{u \rightarrow 0} t(\kappa u)/t(u) = \kappa^{\eta \rho - 1}$, therefore proving the regular variation of $t(u)$ at 0. By using Theorem 1 this in turn proves regular variation of $\bar{P}_e(\rho, N)$ at $N = \infty$. Therefore $\bar{P}_e(\rho, N)$ is both a *c.m.* and a regularly varying function of N for this case. Consequently, when $P_e(\rho x) = \alpha e^{-\eta \rho x}$ and the fading is Rayleigh (i.e. channel gain is exponential), the difference in error rate performance of a MUD system with a random number of users averaged over the number of users distribution and of a deterministic number users approaches zero for sufficiently large λ , as in Theorem 4.

Consider now $P_e(\rho x) = \alpha Q(\sqrt{\eta \rho x})$, with γ_n being exponentially distributed. The error rate can be expressed as,

$$\bar{P}_e(\rho, N) = \alpha \int_0^\infty Q(\sqrt{\eta \rho x}) dF_\gamma^N(x) = \frac{\alpha \sqrt{\eta \rho}}{2\sqrt{2\pi}} \int_0^\infty \frac{e^{N \log(1-e^{-x})} e^{-\eta \rho x/2}}{\sqrt{x}} dx, \quad (19)$$

where the second equality is obtained by integration by parts. Once again, by setting $u = -\log(1 - e^{-x})$ we can rewrite (19) as,

$$\frac{\alpha \sqrt{\eta \rho}}{2\sqrt{2\pi}} \int_0^\infty \exp(-Nu) (1 - e^{-u})^{\eta \rho/2 - 1} \frac{e^{-u}}{\sqrt{-\log(1 - e^{-u})}} du. \quad (20)$$

Thus we have $t(u) = \alpha \sqrt{\eta \rho} (1 - e^{-u})^{\eta \rho/2 - 1} e^{-u} / (2\sqrt{-2\pi \log(1 - e^{-u})})$ and it can be shown that $\lim_{u \rightarrow 0} t(\kappa u)/t(u) = \kappa^{\eta \rho/2 - 1}$, therefore once again proving that $\bar{P}_e(\rho, N)$ is both a *c.m.* and a regularly varying function of N . Having verified the conditions of Theorem 5 for $P_e(\rho x) = \alpha Q(\sqrt{\eta \rho x})$ with γ_n being exponentially distributed, we conclude the tightness of Jensen's inequality as suggested by Theorem 4.

C. A Special Case: Poisson distributed \mathcal{N} and Rayleigh Faded Channels

In this section, we consider the case when the number of users \mathcal{N} is Poisson distributed and the user channels are Rayleigh faded. This practically relevant case will lead to closed form expressions.

1) *Distribution of Channel Gain:* For this case the CDF of the channel gain of the best user using (14) is given by,

$$F_{\gamma^*}(x) = \exp(-\lambda e^{-x}) I[x \geq 0]. \quad (21)$$

The channel gain of the best user in (21) is identical to a truncated Gumbel distribution, which was seen in its untruncated form in (15). Notice that for $x = 0$ (21) yields $e^{-\lambda} > 0$ so $F_{\gamma^*}(x)$ has a jump at $x = 0$. The distribution in (21) is therefore of mixed type with a mass of $e^{-\lambda}$ at the origin and the rest of the distribution has the form of a *truncated Gumbel distribution*.

2) *Average Error Rate:* Assuming the error rate has the form, $P_e(\rho x) = \alpha e^{-\eta \rho x}$ as mentioned in Section III-A, the average error rate can be expressed as,

$$\mathbb{E}_{\mathcal{N}} [\bar{P}_e(\rho, \mathcal{N})] = \lambda \alpha \int_0^\infty e^{-\eta \rho x} e^{-x} e^{-\lambda e^{-x}} dx + \alpha \int_0^\infty \delta(x) e^{-\lambda} e^{-\eta \rho x} dx. \quad (22)$$

Setting $y = \lambda e^{-x}$ and integrating by substitution, (22) can be expressed as,

$$\mathbb{E}_{\mathcal{N}} [\bar{P}_e(\rho, \mathcal{N})] = \alpha \int_0^\lambda \left(\frac{y}{\lambda}\right)^{\eta \rho} e^{-y} dy + \alpha e^{-\lambda} = \alpha \lambda^{-\eta \rho} \gamma(\eta \rho + 1, \lambda) + \alpha e^{-\lambda}, \quad (23)$$

where $\gamma(s, x)$ is the lower incomplete gamma function [12]. It can be easily shown that $\alpha \lambda^{-\eta \rho} \gamma(\eta \rho + 1, \lambda) + \alpha e^{-\lambda} \sim \alpha \lambda^{-\eta \rho} \Gamma(\eta \rho + 1)$ as $\lambda \rightarrow \infty$, indicating a power-law decay in the error rate as a function of the average number of users.

D. Asymptotic Scaling of Capacity with λ

Next, we derive the asymptotic average capacity and the corresponding scaling laws with respect to λ .

Theorem 6: For Poisson distributed \mathcal{N} with mean λ and Rayleigh faded channels, as $\lambda \rightarrow \infty$, we have

$$\mathbb{E}_{\mathcal{N}} [\bar{C}(\rho, \mathcal{N})] = \log(1 + \rho \log(\lambda)) + O(1/\sqrt{\log(\lambda)}). \quad (24)$$

Proof: See Appendix B. ■

For a MUD system with deterministic number of users N , it has been shown in [1] that the ergodic capacity grows as $\log \log(N)$. From Theorem 6 it is seen that for a MUD system with random number users, whose mean is λ , the ergodic capacity grows as $\log \log(\lambda)$. This implies that when average number of users λ is equal to N of the deterministic number of users case, the ergodic capacity for both cases grow at the same rate.

E. Zero Truncated Poisson User Distribution

The CDF expression in (21) includes the case when $N = 0$, i.e., there are no users in the system. When there are no users, no data will be transmitted. In view of this, it is reasonable to drop the $N = 0$ case and model the user distribution with the *zero-truncated Poisson distribution* which is given by $\Pr[\mathcal{N} = k | \mathcal{N} > 0] = \Pr[\mathcal{N} = k] / (1 - \Pr[\mathcal{N} = 0])$, for any positive integer k . For zero-truncated Poisson distributed \mathcal{N} , $N \in \{1, 2, \dots\}$ and mean $\underline{\lambda} = \lambda / (1 - e^{-\lambda})$ where λ is the mean of the underlying Poisson random variable. The CDF of the channel gain of the best user can be expressed as,

$$F_{\gamma^*}(x) = \frac{1}{1 - e^{-\lambda}} \sum_{k=1}^{\infty} [F_{\gamma}(x)]^k \underline{\lambda}^k \frac{e^{-\lambda}}{k!} = \frac{e^{\lambda(1-e^{-x})} - 1}{e^{\lambda} - 1}. \quad (25)$$

For when the average number of users $\lambda \rightarrow \infty$, it can be seen that $\underline{\lambda} \sim \lambda$. Further,

$$\lim_{\lambda \rightarrow \infty} F_{\gamma^*}(x) = \lim_{\lambda \rightarrow \infty} \frac{e^{\lambda(1-e^{-x})} - 1}{e^{\lambda} - 1} = e^{-\lambda(e^{-x})}, \quad (26)$$

which is identical to the CDF for Poisson distributed user case in (21). This implies that for a large average number of users in the system, the outage, average error rate, and ergodic capacity performance of zero-truncated Poisson distributed user case will be identical to that of the Poisson distributed user case.

VII. SIMULATIONS

An uplink MUD system where both the BS and users having a single antenna is considered. In this section, using Monte-Carlo simulations, the error rate, ergodic capacity and outage capacity are simulated to corroborate our analytical results. For all simulations considered, the Rayleigh fading is assumed.

In Figure 1, assuming $\pi/4$ QPSK modulation, the average bit error rate with deterministic N is compared with the performance averaged across various user distributions. It is seen that the deterministic number of users system performs better than all the cases involving random number of users. The performance of the Poisson distributed users case comes close to the deterministic case as λ increases, as predicted by Theorem 4.

In Figure 2, the ergodic capacity is plotted against λ for the random cases and $N = \lambda$ for the deterministic case. It is seen that the capacity of the deterministic number of users system is the highest while for all distributions of \mathcal{N} , the capacity is worse, corroborating our result in Section III-B.

In Section IV, we showed that Poisson distributed random variables and geometric distributed random variables are LT ordered, which also orders their respective average error rate and ergodic capacities when averaged across the respective user distributions. In Figures 3 and 4 it can be seen that both error rate and capacity follow their corresponding ordering at all average SNR ρ .

In Figure 5 the bit error rate versus average SNR for different average number of users is shown. It can be seen that the analytical approximation of average error rate with Poisson number of users derived in (23) is within 1 dB of the Monte-Carlo simulation result. Following the result in (23) it can also be seen that the larger the value of λ , the lower the error rate.

In previous simulations we saw that increasing λ leads to an improvement in performance, and for a fixed average SNR the performance of the system with Poisson distributed number of users approaches the performance of the system with deterministic number of users. Figure 6 considers a zero-truncated Poisson distribution and illustrates that at high average SNR, the diversity order is $k_0 d = 1$, verifying Theorem 3. This leads us to conclude that for low SNR's

but sufficiently large λ the performance of the random number of users is nearly identical to that of the deterministic case. However for high SNR's, the performance of the random number of users case is significantly worse due to the loss in diversity order.

VIII. CONCLUSIONS

Multi-user diversity (MUD) is analyzed for when the number of users in the system is random. The error rate of MUD systems is proved to be a completely monotone function of the number of users in the system, which also implies convexity. Further, ergodic capacity is shown to have a completely monotone derivative with respect to the number of users. Using Jensen's inequality, it is shown that the average error rate and ergodic capacity averaged across fading *and* the number of users will always perform inferior to the corresponding performance of a system with deterministic number of users. Further, we provide a method to compare the performance of the system for different user distributions, using a specific stochastic ordering based on the Laplace transform of user distributions.

Importantly, for the MUD system with random number of users, it is shown that the diversity order is defined by the minimum of the range of realizations of the number of users. When the number of users are Poisson distributed, for any user channel fading distribution, outage probability is shown to converge to the truncated Gumbel CDF, similar to the case of the deterministic number of users system. Further, it is proved that the difference between the error rate performance of the Poisson number of users system and the deterministic number of users case goes to zero like $O(\bar{P}_e(\rho, \lambda)/\lambda)$ asymptotically in the average number of users. As a special case, when the user fading channels are Rayleigh distributed, a closed-form error rate expression is provided. Also, the asymptotic scaling law of ergodic capacity is analyzed and shown to be approximately $\log(1 + \rho \log(\lambda))$ for large λ . Finally, zero-truncated Poisson number of users case is shown to not affect our main conclusions for the common scenario where the number of users is always positive.

APPENDIX A. PROOF OF THEOREM 3

Since $F_\gamma(x)$ is regularly varying, it must be in the form $F_\gamma(x) = x^d l(x)$ where $l(x)$ is slowly varying at 0. For a system with k users, the CDF of the channel gain becomes $F_\gamma^k(x) = x^{kd} l^k(x)$. It is easy to verify that $l^k(x)$ is slowly varying at 0. Therefore, given $\int_0^t dF_\gamma^k(x) \sim t^{kd} l^k(t)$ and $P_e(\rho x) = \alpha e^{-\eta \rho x}$, it follows based on Theorem 1 that

$$\bar{P}_e(\rho, k) = \alpha \int_0^\infty e^{-\eta \rho x} dF_\gamma^k(x) \sim \alpha \Gamma(kd + 1) F_\gamma^k(\eta^{-1} \rho^{-1}) \quad (27)$$

as $\rho \rightarrow \infty$. For the case $P_e(\rho x) = \alpha Q(\sqrt{\eta \rho x})$, the asymptotic expression of $\bar{P}_e(\rho, k)$ can be derived similarly as follows. Using integration by parts we obtain

$$\bar{P}_e(\rho, k) = \alpha \int_0^\infty Q(\sqrt{\eta \rho x}) dF_\gamma^k(x) = \frac{\alpha \sqrt{\eta \rho}}{2\sqrt{2\pi}} \int_0^\infty \frac{e^{-\eta \rho x/2}}{\sqrt{x}} F_\gamma^k(x) dx \quad (28)$$

Based on (5) we have

$$\int_0^t \frac{1}{\sqrt{x}} F_\gamma^k(x) dx \sim l^k(t) \int_0^t x^{kd-1/2} dx = \frac{1}{kd + 1/2} t^{kd+1/2} l^k(t) \quad (29)$$

as $t \rightarrow 0$. Then the asymptotic average error rate given by (28) becomes

$$\begin{aligned} \bar{P}_e(\rho, k) &\sim \frac{\alpha \sqrt{\eta \rho}}{(2kd + 1)\sqrt{2\pi}} \left(\frac{\eta \rho}{2}\right)^{-kd-1/2} \Gamma(kd + 3/2) l^k(2\eta^{-1} \rho^{-1}) \\ &= \frac{\alpha \Gamma(kd + 1/2)}{2\sqrt{\pi}} \left(\frac{\eta \rho}{2}\right)^{-kd} l^k(2\eta^{-1} \rho^{-1}) = \frac{\alpha \Gamma(kd + 1/2)}{2\sqrt{\pi}} F_\gamma^k(2\eta^{-1} \rho^{-1}) \end{aligned} \quad (30)$$

as $\rho \rightarrow \infty$, where the asymptotic equality is based on Theorem 1 and the second equality is based on $F_\gamma(x) = x^d l(x)$. It can be seen that both (27) and (30) have the form $\bar{P}_e(\rho, k) \sim C_1 F_\gamma^k(C_2 \rho^{-1})$ with C_1, C_2 being constants. Consequently, as $\rho \rightarrow \infty$, the dominant term in the average error rate $E_{\mathcal{N}} [\bar{P}_e(\rho, \mathcal{N})] = \sum_{k=k_0}^\infty \Pr[\mathcal{N} = k] \bar{P}_e(\rho, k)$ is the term with $k = k_0$, which decreases slower than any other term. Using dominated convergence theorem, we can easily determine $\lim_{\rho \rightarrow \infty} E_{\mathcal{N}} [\bar{P}_e(\rho, \mathcal{N})] / \bar{P}_e(\rho, k_0) = \Pr[\mathcal{N} = k_0]$ by exchanging limit and summation. Consequently, the ratio between the left hand side and the right hand side of (13) goes to 1 as $\rho \rightarrow \infty$, and we thus have the asymptotic average error rate given by (13), with C_1, C_2 given

as in the theorem.

APPENDIX B. PROOF OF THEOREM 6

For Poisson distributed \mathcal{N} and Rayleigh faded channels, using integration by parts and the CDF in (21), the capacity of the system can be written as,

$$\mathbb{E}_{\mathcal{N}} [\bar{C}(\rho, \mathcal{N})] = \rho \int_0^\infty \frac{1 - e^{-\lambda e^{-x}}}{1 + \rho x} dx. \quad (31)$$

Defining $y := e^{-x}$ and integrating by substitution,

$$\begin{aligned} \mathbb{E}_{\mathcal{N}} [\bar{C}(\rho, \mathcal{N})] &= \int_0^1 \frac{1 - e^{-\lambda y}}{1 - \rho \log(y)} \left(\frac{\rho}{y} \right) dy \\ &= \int_0^{\sqrt{\log(\lambda)}/\lambda} \frac{1 - e^{-\lambda y}}{1 - \rho \log(y)} \left(\frac{\rho}{y} \right) dy + \int_{\sqrt{\log(\lambda)}/\lambda}^1 \frac{\rho(1 - e^{-\lambda y})}{y(1 - \rho \log(y))} dy. \end{aligned} \quad (32)$$

For the first term after the second equality in (32), we have

$$\begin{aligned} 0 &< \int_0^{\sqrt{\log(\lambda)}/\lambda} \frac{1 - e^{-\lambda y}}{1 - \rho \log(y)} \left(\frac{\rho}{y} \right) dy < \int_0^{\sqrt{\log(\lambda)}/\lambda} \frac{\lambda y}{1 + \rho \log(\lambda) - (\rho/2) \log(\log(\lambda))} \left(\frac{\rho}{y} \right) dy \\ &= \frac{\rho \sqrt{\log(\lambda)}}{1 + \rho \log(\lambda) - (\rho/2) \log(\log(\lambda))}, \end{aligned} \quad (33)$$

by replacing the numerator of the integrand with its upper bound and the denominator of the integrand with its lower limit. It can be seen that the upper bound after the equality in (33) yields $O(1/\sqrt{\log(\lambda)})$ and has limit 0 as $\lambda \rightarrow \infty$, implying that the first term should have limit 0. The second term in (32) has the bounds given by,

$$\int_{\sqrt{\log(\lambda)}/\lambda}^1 \frac{\rho(1 - e^{-\sqrt{\log(\lambda)}})}{y(1 - \rho \log(y))} dy < \int_{\sqrt{\log(\lambda)}/\lambda}^1 \frac{\rho(1 - e^{-\lambda y})}{y(1 - \rho \log(y))} dy < \int_{\sqrt{\log(\lambda)}/\lambda}^1 \frac{\rho(1 - e^{-\lambda})}{y(1 - \rho \log(y))} dy \quad (34)$$

in which the lower and upper bounds are obtained by bounding the numerator, and they turn out to be $(1 - e^{-\sqrt{\log(\lambda)}}) \log(1 + \rho \log(\lambda) - (\rho/2) \log(\log(\lambda)))$ and $(1 - e^{-\lambda}) \log(1 + \rho \log(\lambda) - (\rho/2) \log(\log(\lambda)))$ respectively. Also, it can be verified that $\log(1 + \rho \log(\lambda) - (\rho/2) \log(\log(\lambda))) =$

$\log(1 + \rho \log(\lambda)) + O(\log(\log(\lambda))/\log(\lambda))$ as $\lambda \rightarrow \infty$, and $\lim_{\lambda \rightarrow \infty} e^{-\sqrt{\log(\lambda)}} \log(1 + \rho \log(\lambda)) = \lim_{\lambda \rightarrow \infty} e^{-\lambda} \log(1 + \rho \log(\lambda)) = 0$. Therefore, for a fixed ρ , and as $\lambda \rightarrow \infty$ we can express (32) as (24) considering the fact that $\log(\log(\lambda))/\log(\lambda)$ decays faster than $1/\sqrt{\log(\lambda)}$, completing the proof.

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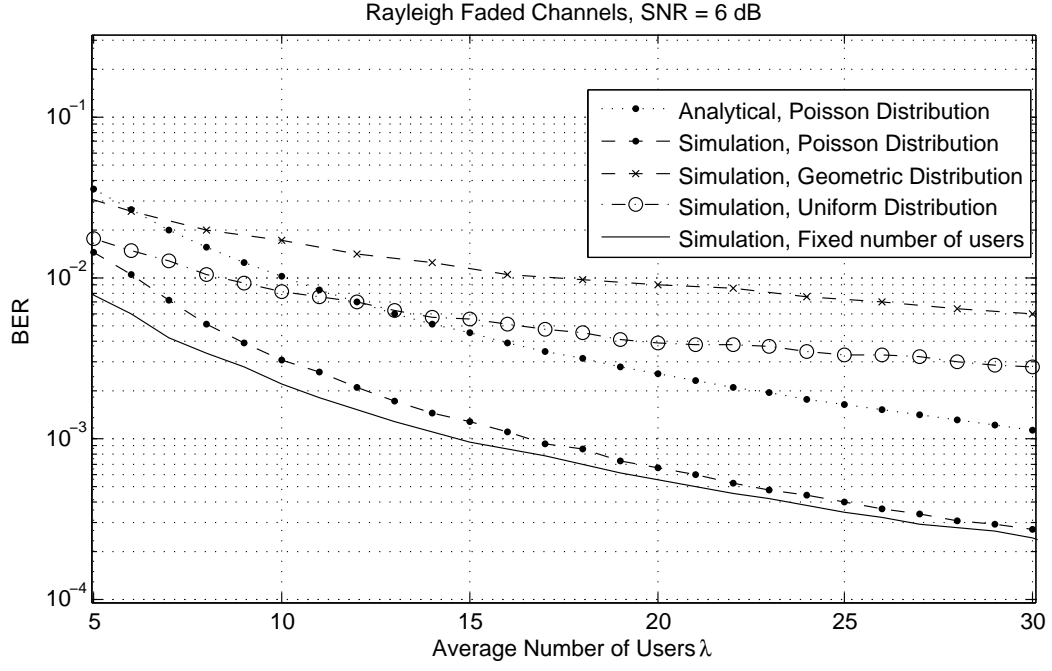


Fig. 1. Error rate vs. λ : Rayleigh Fading Channel, average SNR = 6 dB

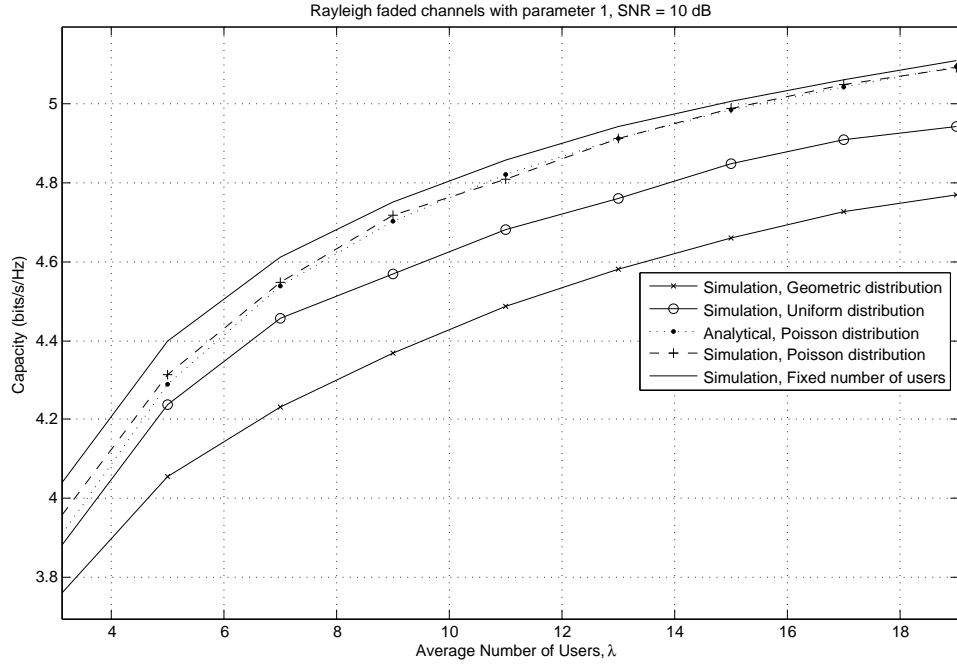


Fig. 2. Capacity vs. λ : Rayleigh Fading Channel, average SNR = 10dB

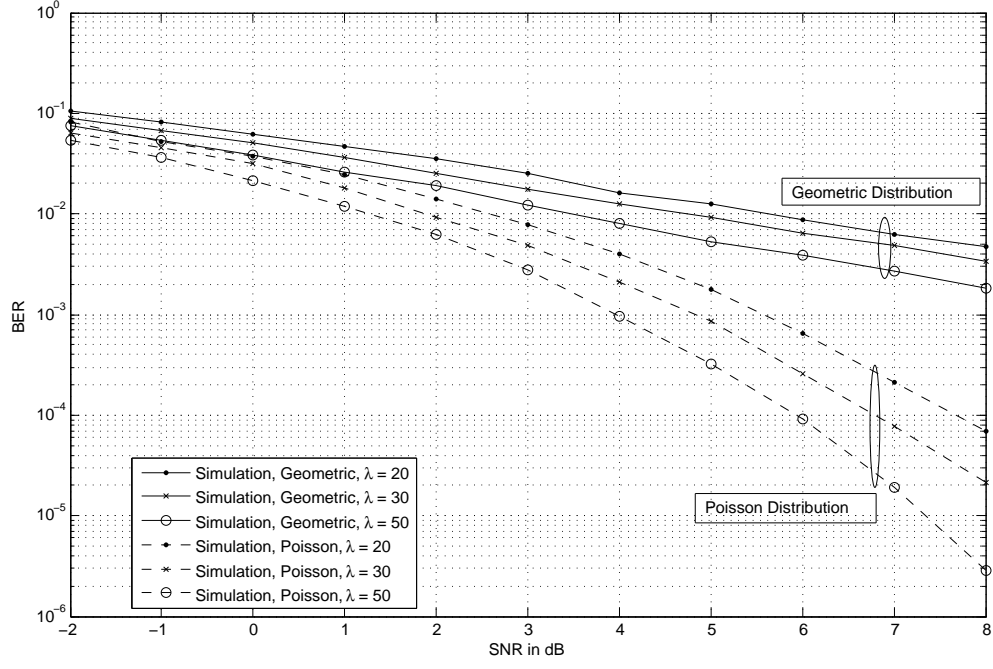


Fig. 3. Error rate vs. average SNR: Rayleigh Fading Channel

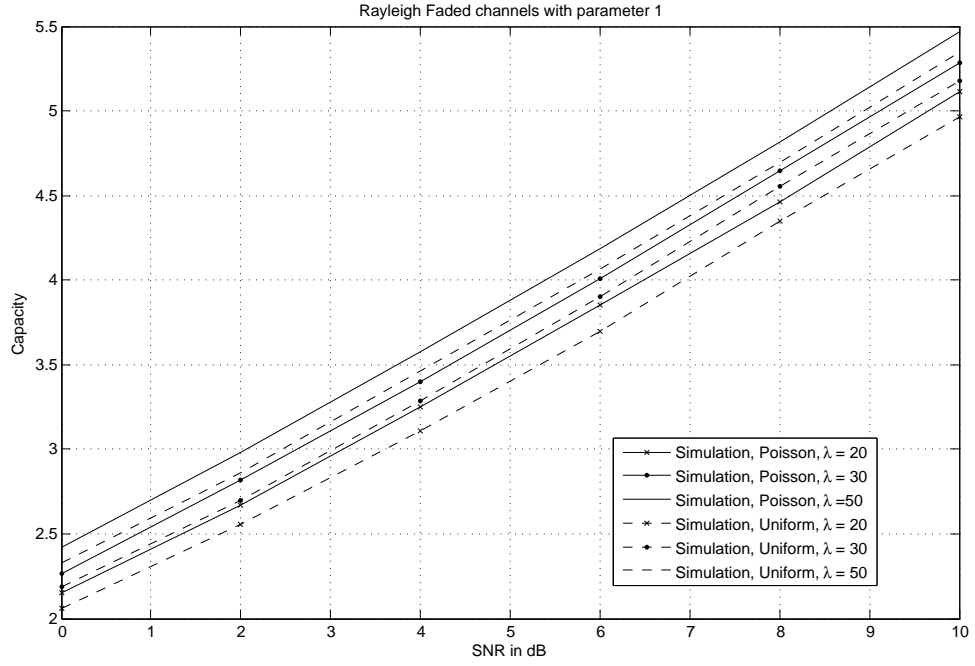


Fig. 4. Capacity vs. average SNR: Rayleigh Fading Channel

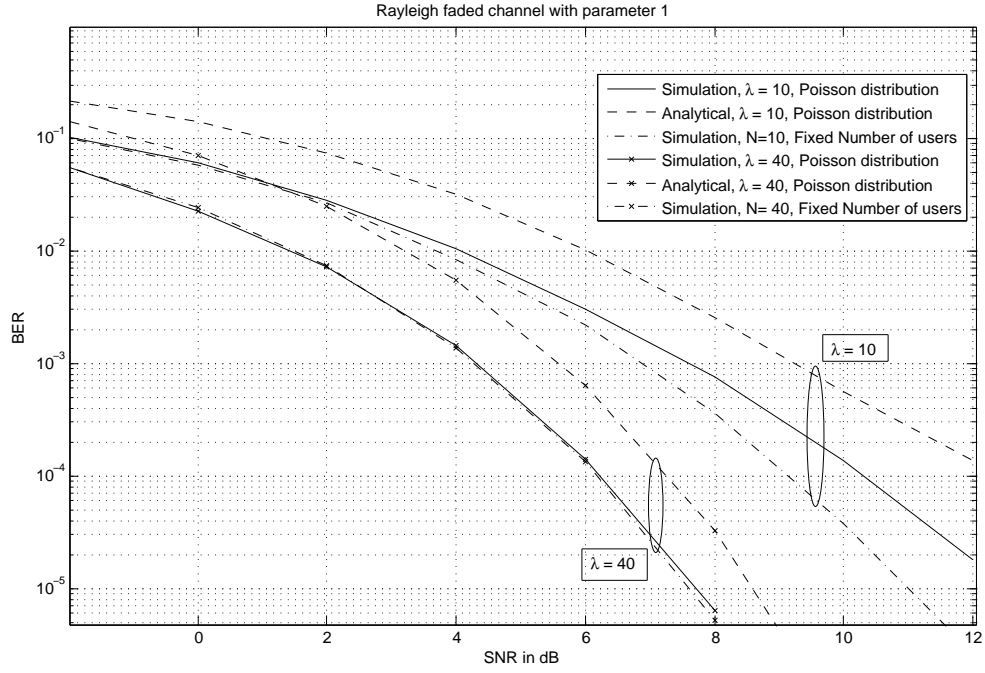


Fig. 5. Error rate vs. average SNR: Poisson Users and Rayleigh Fading Channel

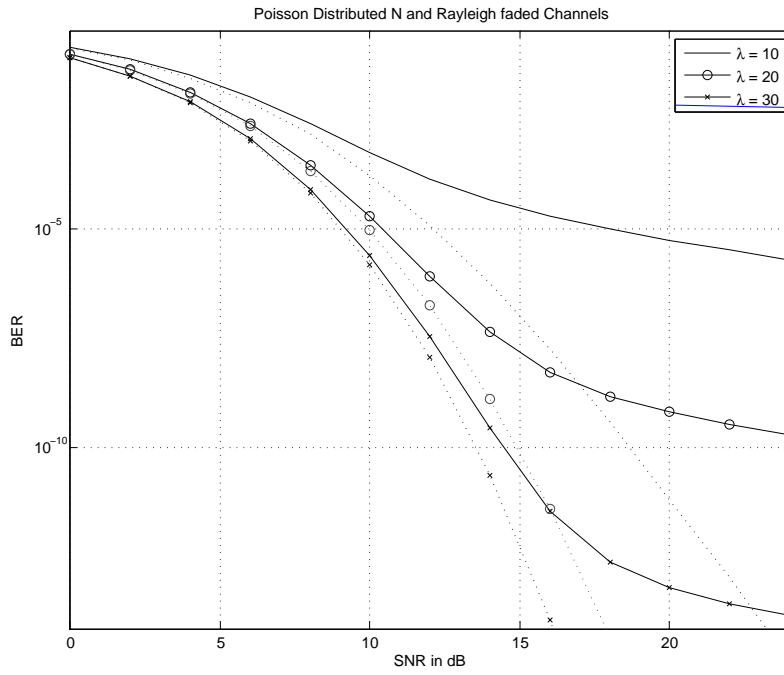


Fig. 6. Diversity Analysis: Poisson Users and Rayleigh Fading Channel